

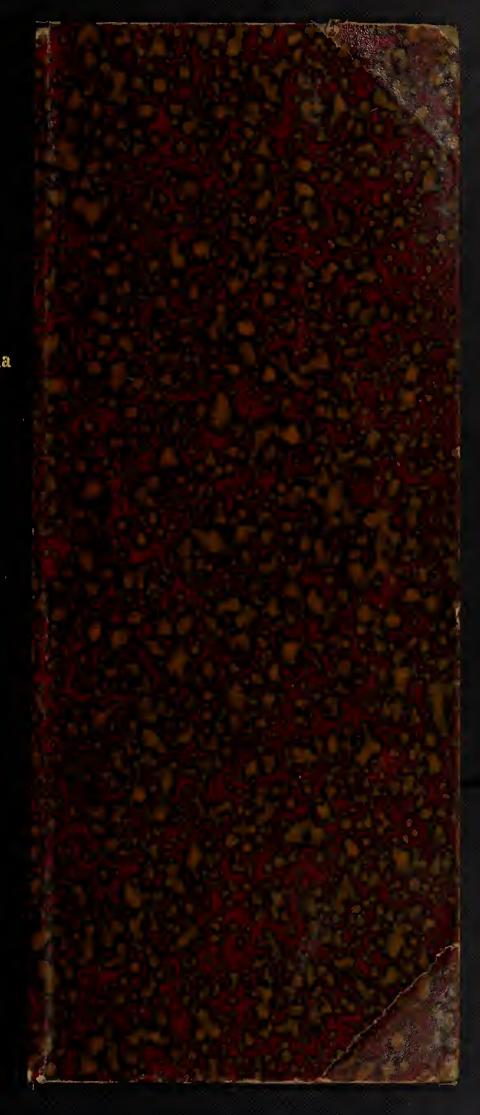
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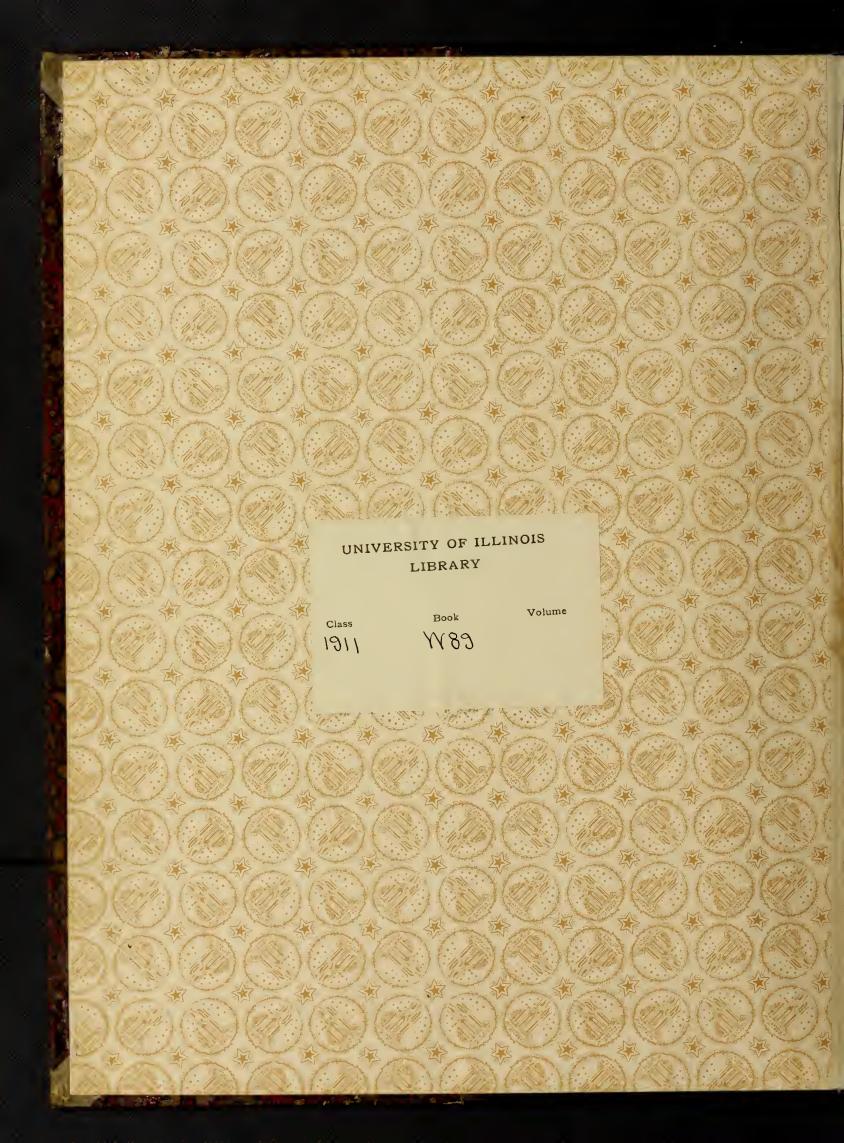
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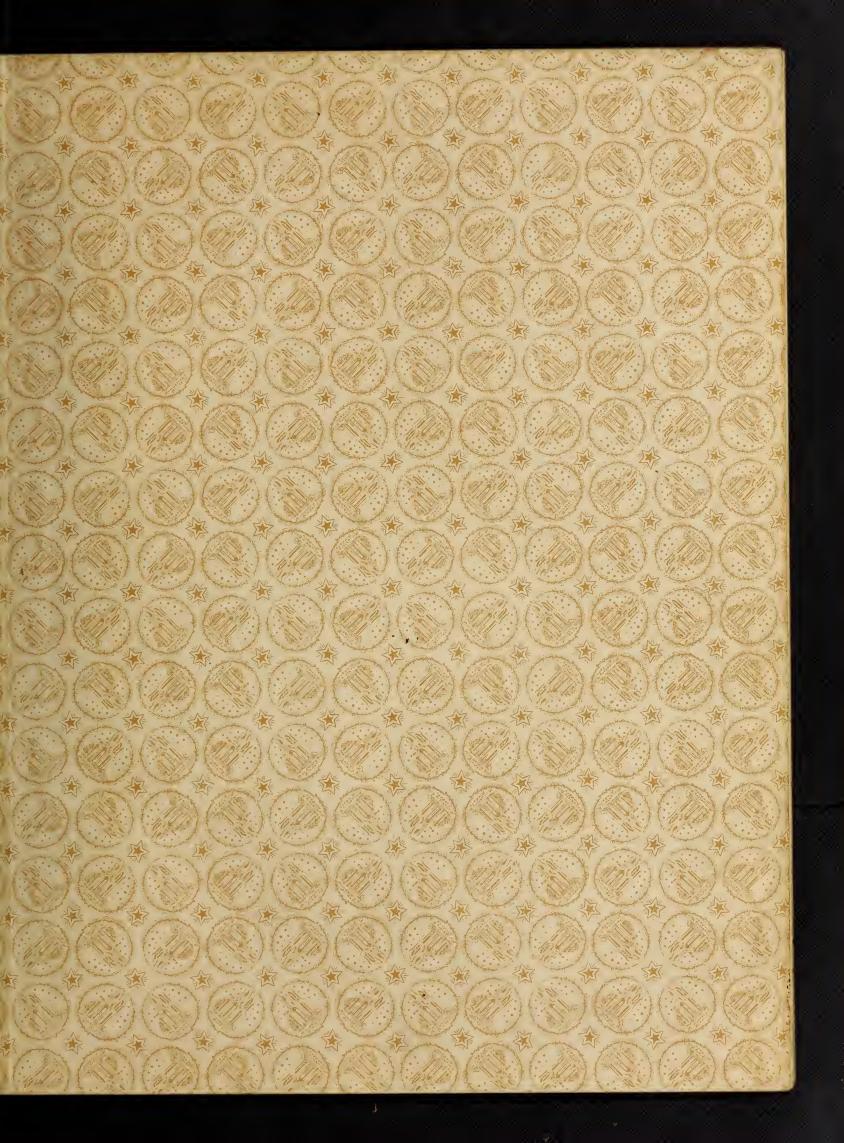
Transformer in Transient Phenomena

Physics

M.S.









LIMITATIONS OF THE SERIES TRANSFORMER

IN

TRANSIENT PHENOMENA

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BY

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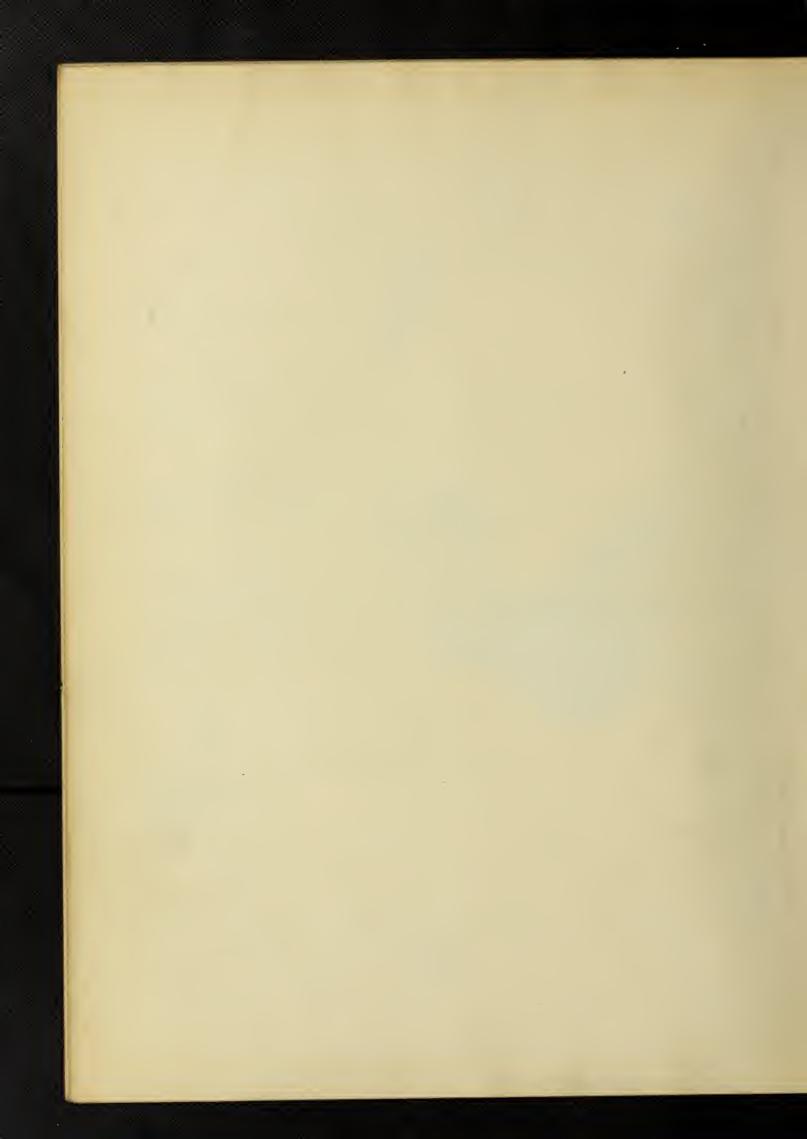
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LIMITATIONS OF THE SERIES TRANSFORMER IN TRANSIENT PHENOMENA.

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LIMITATIONS OF THE SERIES TRANSFORMER IN TRANSIENT PHENOMENA.

INTRODUCTION.

The following work is the outgrowth of a series of tests and calculations, made on the short circuit currents of alternators. A number of oscillograph records were taken of the short circuit current, and the instantaneous values compared with those calculated from the constants of the alternator. The agreement was close, in tests made in the laboratory, where the oscillograph was direct connected to the system; but very considerable discrepancies were found in some tests made outside of the laboratory, where the oscillograph was connected in the secondary circuit of a series transformer. Since the theory of the short circuit currents of alternators would not explain the peculiaraties in the latter curves, a study of the action of series transformers on unsymetrical or transient currents, was undertaken in an attempt to explain the phenomena. The results of this investigation shows, that the series transformer(especially with iron core) is unreliable for recording transient or unsymetrical currents. While there: is really only one type of current transformer in practical use, which type has an iron magnetic circuit, it is of interest to consider the action of such a transformer without iron.

AIR CORE SERIES TRANSFORMER.

The air core type will be discussed first, since its operation can be expressed by a mathematical equation, and thus clear insight into the whole problem can readily be given.

A series transformer is a mutual inductance, where the effect of the secondary circuit upon the primary current isso small as to be negligible. Since the secondary circuit is closed upon itself, the differential equation of the circuit becomes,

where r", x" and i" are the resistance, reactance and current in the secondary circuit; X is the mutual inductive reactance between the primary and secondary circuits; and i' is the primary current. As stated above, i' is not effected by i" and therefore di'/de is not a function of i", and the single differential equation is sufficient for the solution of the problem.

If the primary current i' contains a transient term, as the starting alternating current in an inductive circuit,

Differentiating equation 2

$$di'/de = A(\cos(e-s) + (a)\sin(e,-s)e^{-ae'}) - - - - - 3.$$

^{*} Steinmetz Electric Phenomena and Oscillations. p 43.

- 1 - 1 + al

Equation 3 substituted in equation 1 gives

$$x^{n}di^{n}/de + r^{n}i^{n} = -XA(cos(e-s) + (a)sin(e,-s)e^{-ae^{i}})-- 4.$$

Equation 4 is easily written in the familiar form

$$di''/de' + bi'' = -XA/x''(cos(e-s) + (a)sin(e, -s)e^{-ae'}) -- 5.$$

Where the factor r"/x" is replaced by b.

The solution of equation 5 is

$$i'' = -A \frac{X}{X} e^{-b\theta} \left(\int \cos(\theta - \theta) e^{b\theta'} + (a) \sin(\theta - \theta) e^{(b-a)\theta'} + c \right)$$

$$= -A\frac{X}{x''} \left(\frac{\sin(e-s+B)}{\sqrt{(b^2+1)}} + \frac{a}{b-a} \sin(e-s)e^{-ae^{t}} + Ce^{-be^{t}} \right) - -- 6.$$

Where B = arc tan b.

When $e = e_1$, that is when e' = 0, i' = 0 and i'' = 0.

Therefore

$$C = \frac{\sin(e_{1}-s+B)}{\sqrt{b^{2}+1}} + \frac{a}{b-a}\sin(e_{1}-s) - \frac{b}{b-a} - \frac{b}{b^{2}+1}\sqrt{\frac{b+1}{b-a}+1}\sin(e_{1}-s+p) - \frac{a}{b-a} = \frac{b}{b^{2}+1}\sqrt{\frac{b+1}{b-a}+1}\sin(e_{1}-s+p) - \frac{a}{b-a} = \frac{b}{b^{2}+1}$$

Where $p = arc tan \frac{b-a}{b+1}$.

Knowing the instantaneous values of i',i" and the ratio of the secondary to primary turns n"/n', the instantaneous value of magnetizing current i ds given by the following equation,

Substituting equations 6 and 2 in 9

$$i = A \left[\sin(e-s) - \sin(e,-s)e^{-ae^{t}} - \frac{Xn^{t}}{x^{t}n^{t}} \left(\frac{\sin(e-s+B)}{b^{t}+1} + \frac{a}{b-a}\sin(e,-s)e^{-ae^{t}} + Ce^{-be^{t}} \right) \right] - 10.$$

* Murrey Differential Equations. p 27.

Representing X n" by the constant k and replacing and combining the the trigonometric functions into one, the

equation becomes $i = A \left\{ \sqrt{(1 - \frac{2k - k^2}{b^2 + 1})} \sin(e - s - u) - \frac{ks}{b - a}} \sin(e - s) e^{-ae'} - \text{Ce}^{-be'} \right\} 11$

Where $u = arc \tan \frac{kb}{b^2+1}$, which is the angle of lag of the stable magnetizing current behind the primary current.

Example 1.

As an example of the application of this method may be considered the following case.

$$a = r^{1}/x^{1} = .1$$
; $b = r^{1}/x^{2} = .01$; $X = 10.$; $x^{2} = 100.$; $A = 1.$
 $s = 85^{\circ}$; $e_{1} = 175^{\circ}$; and $n^{1}/n^{1} = 9.5$.

Substituting these constants in equation 2, - - -

 $p = arc tan \frac{b-a}{b+1} = arc tan.0899 = 5°.$

The value of C from equation & becomes

 $C = .1119 \rightarrow / 1$

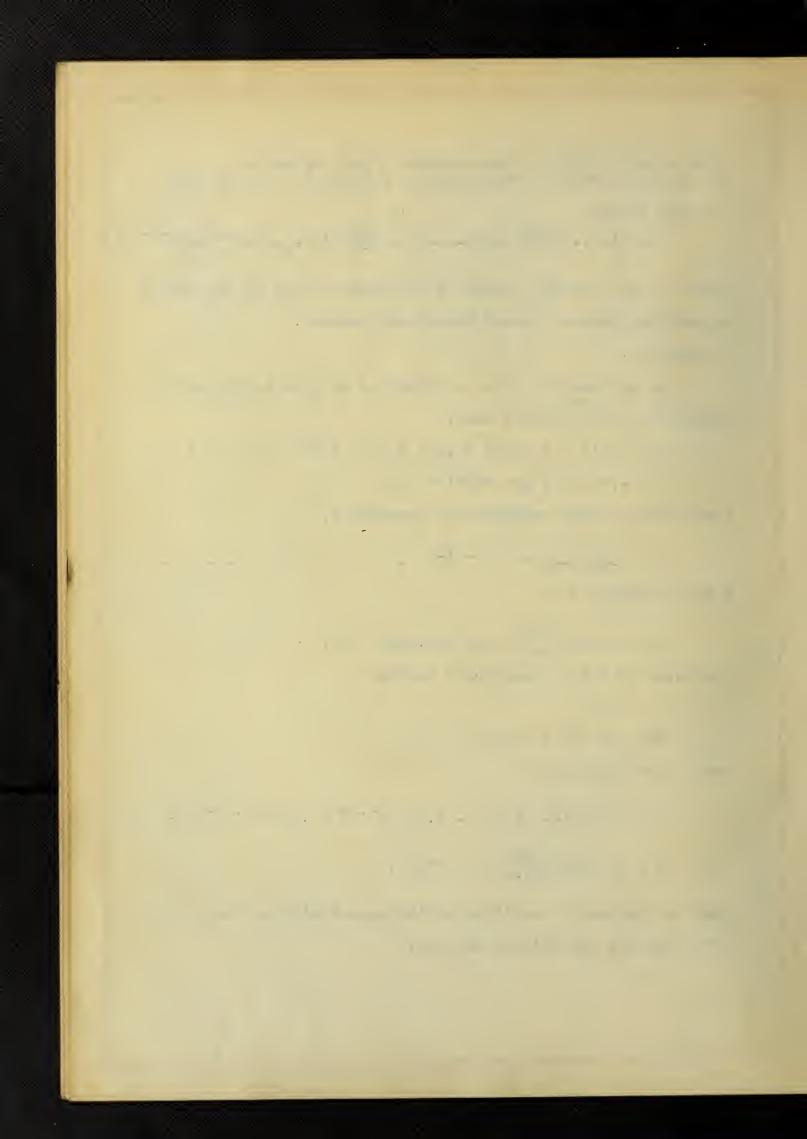
B = arc tan b = 35'.

Hence from equation 6

$$i'' = -.1 \left\{ \sin(e - 174.4^{\circ}) - 1.1111e^{-.16'} + .1119e^{-.016'} \right\} - B'$$

$$u = \arctan \frac{kb}{b^2 + 1 - k} = 10^{\circ} 45'.$$

That is the ztable condition of the magnetizing current lags 10°45' behind the primary current.



i from equation 11 becomes

equation it becomes $1 = -\left(.05 \sin(e^{-1}\%5.75^{\circ}) + .055e^{-.16^{\circ}} - .1055e^{-.016^{\circ}}\right) \text{ C'}.$ On the opposite page equations A', B' and C' are plotted with e as abscissae and i',i" and i as ordinates. The ordinates derived from equation B' are multiplied by the transformation ratio $x^n/x = 10$ and turned 180° so as to be superimposed upon the primary current for better comparison. The curves thus plotted show clearly the error in secondary current due to the air core transformer with a 5% magnetizing current.

As a second problem a primary current which is unsymmetrical might be considered. If the current lies entirely above the zero line, the equation would be

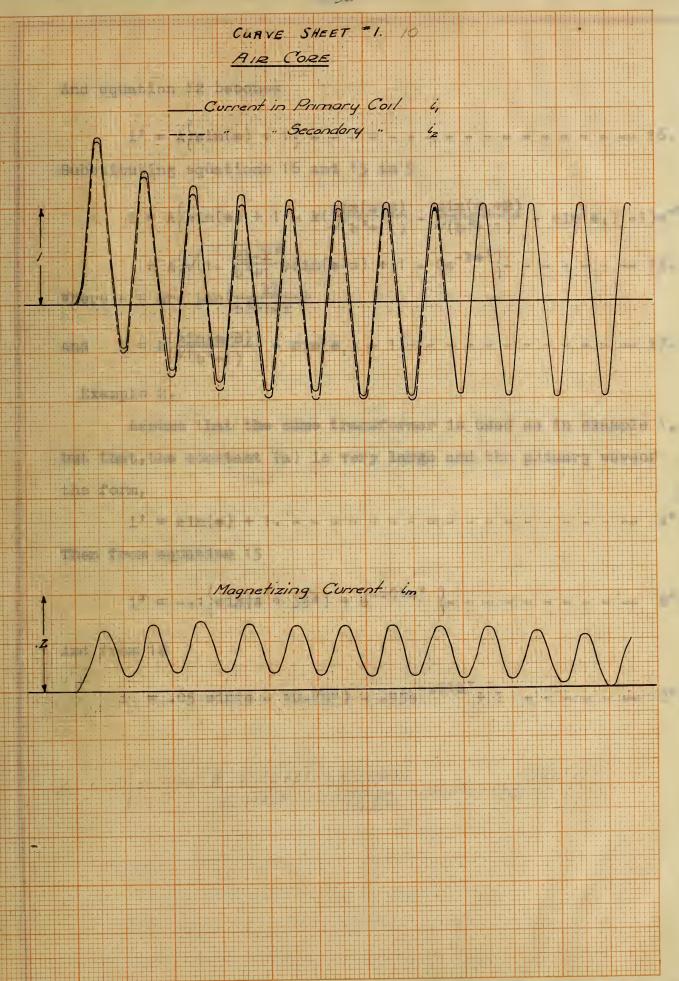
Differentiating 12 and substituting in equation 1

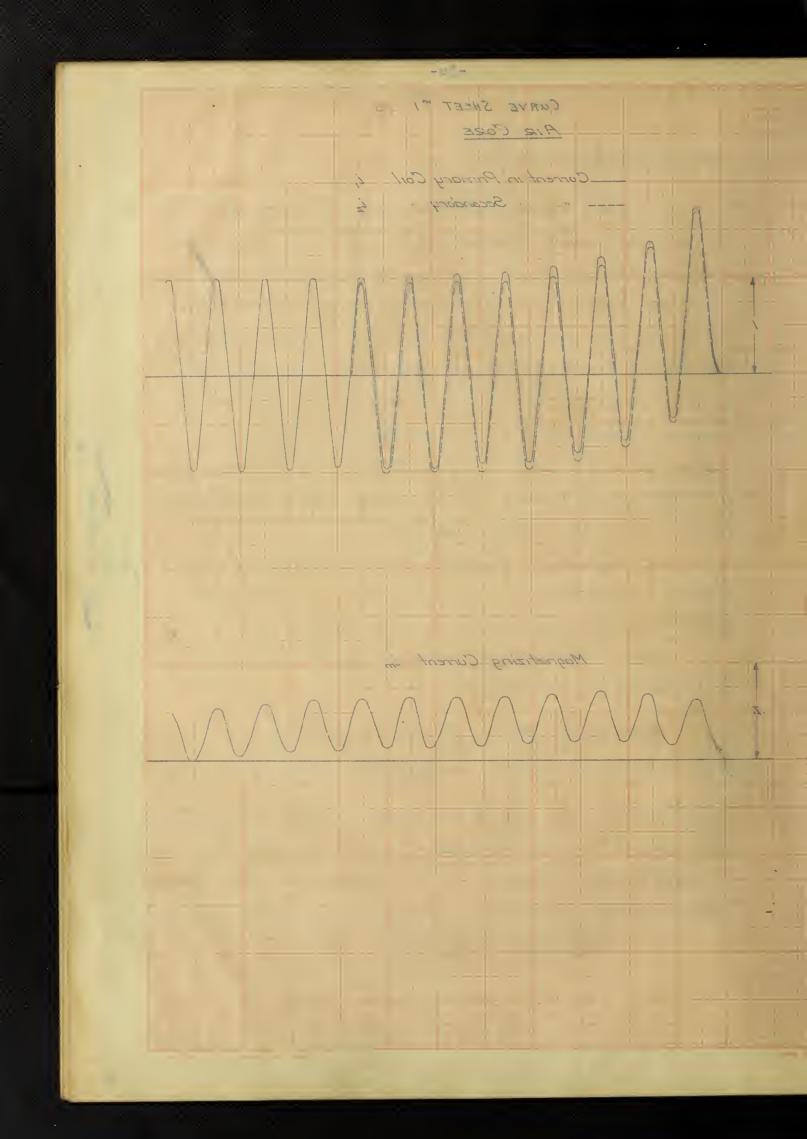
 $di^{\dagger}/de + bi^{\dagger} = -A\left\{\frac{X}{x^{\dagger}}\right\}\left\{\cos(e) + Cae^{-ae^{\dagger}}\right\} = -$ And the solution of equation 13 is

$$i'' = -A\frac{X}{X''} \frac{(\sin(e+B))}{(\sqrt{b^2+1})} + \frac{aC}{b-a} e^{-ae^2} + C^2 e^{-be^2} - - - - = -14.$$
Where B = arc tan b and C' = $-\left\{\frac{\sin(e+B)}{\sqrt{b^2+1}} + \frac{aC}{b-a}\right\}$.

If the time constant 1/a of the primary circuit is exceedingly small, then e-as' will practically be zero in a very short time; (a) will be large in comparison to (b) and equation 14 becomes, after an infinitesmal time

in infinitesmal time
$$1'' = -A_{X''}^{X} \left[\frac{\sin(e+B)}{\sqrt{b^2+1}} - \frac{(\sin(e+B) - \sin(e) - 1)}{\sqrt{b^2+1}} - \frac{15}{\sqrt{b^2+1}} \right] - 15.$$





And equation 12 becomes

$$i^{\dagger} = A \{ \sin(e) + 1 \}$$
 ---- 16.

Substituting equations 16 and 15 in 9

Example 2.

Assume that the same transformer is used as in example 1, but that, the constant (a) is very large and the primary waveof the form,

i' = sin(e) + 1. - - - - - - A''.

Then from equation 15

$$i^* = -... \{ sin(e + 35^*) + e^{-.01e^*} \}$$

And from 18

1 1 1

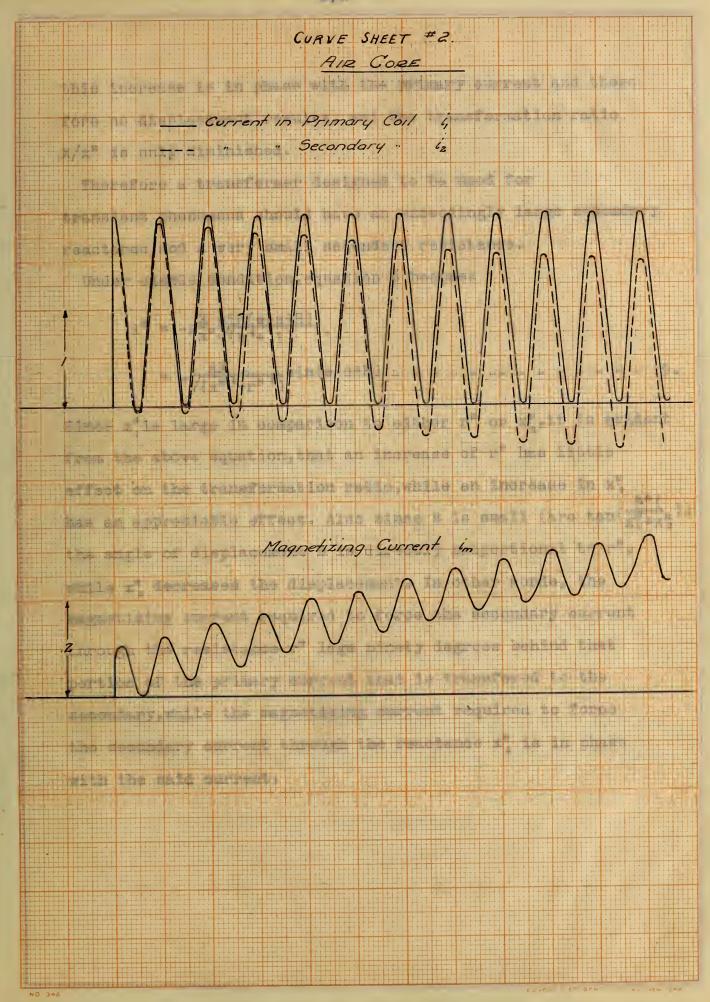
The curves on the opposite page plotted from equations

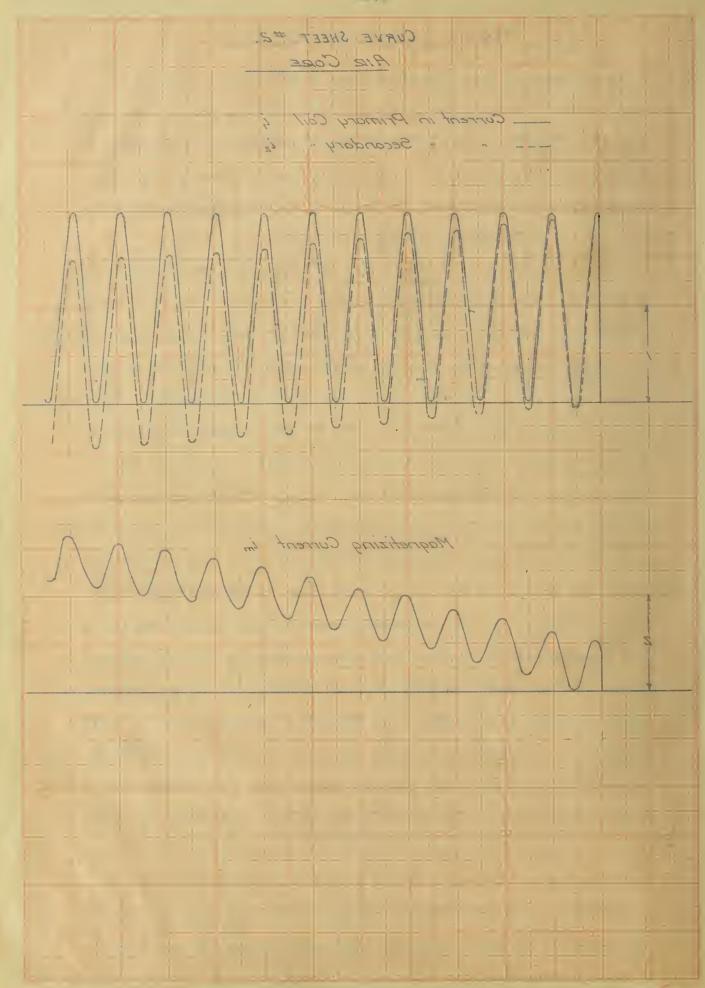
A",B" and C" show clearly how the current in the secondary of
the transformer gradually becomes symetrical in reference to
the zero line, while the magnetizing current creeps up to a
line which is symetrical in respect to the primary current.

DISCUSSION OF THE CONSTANTS

The reactance of the secondary x contains two factors x and x" of which x" represents that portion of the flux that interlinks with the primary coil, and x" represents that portion of the flux that does not interlink with the primary coil, or known as leakage reactance. x, and X hold a definite relation to each other, which ratio, is that of the secondary turns n" to the primary turns n'. The transformation ratio of the transformer is that ratio of the mutual inductive reactance X to the total secondary self inductive reactance x", which ratio can only be considered equal to the ratio of the respective turns in so far as x can be neglected in comparison to x. The resistance in the secondary circuit has little effect on the transformation ratio, but does produce a displacement in phase in the stable condition, and introduces large errors in the transient term, since b is directly proportional to r. The smaller the factor b, the more closely the secondary current follows the primary current, and therefore, for observing transient currents the secondary leakage reactance improves the accuracy; while the resistance, though small, introduces large errors. The value of x is only limited in so far as its effect upon the primary current can be neglected. An increase of x does increase the magnetizing current, but

(* - 9-•, o_i





this increase is in phase with the primary current and therefore no displacement results, and the transformation ratio X/x^n is only diminished.

Therefore a transformer designed to be used for transient phenomena should have an exceedingly large secondary reactance and a very small secondary resistance.

Under stable condition, equation 6 becomes

Since x"is large in comparison to either r" or x", it is evident from the above equation, that an increase of r" has little effect on the transformation ratio, while an increase in x" has an appreciable effect. Also since B is small (arc tan(x"+x")) the angle of displacement B is directly proportional to r", while x" decreases the displacement. In other words, the magnetizing current required to force the secondary current through the resistance r" lags ninety degrees behind that portion of the primary current that is transfered to the secondary, while the magnetizing current required to force the secondary current through the reactance x" is in phase with the said current.

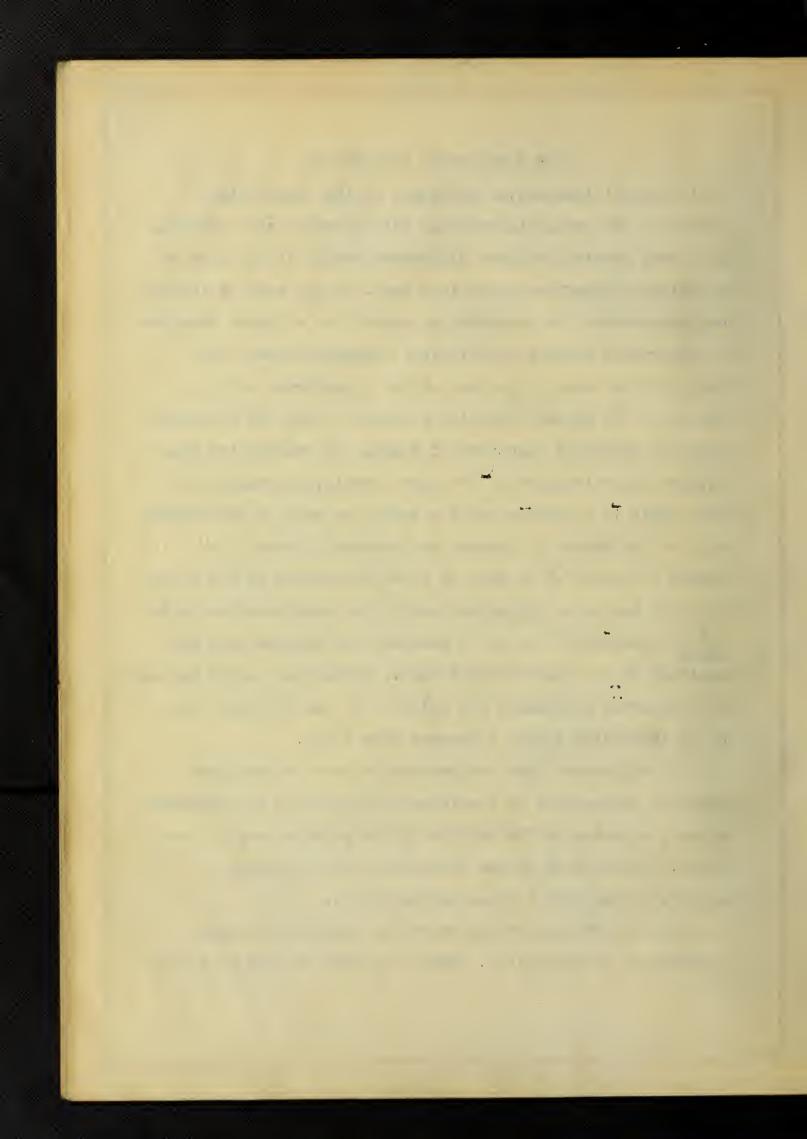
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IRON CORE SERIES TRANSFORMER

This gradual increase, or greeping, of the magnetizing current of the series transformer with unsymetrical currents, has a much greater and more disastrous effect in the case of an ordinary transformer with iron core. In the case of the air core transformer the magnetizing current was a direct function of the primary current in all stable conditions; but this does not hold true in the case of the transformer with an iron core. The mutual inductive reactance X and the correspon ding self inductive reactance x, depend, in exactly the same way, upon the reluctance of the core circuit, and consequently their ratio is a constant, with a value the same as that of the ratio of the number of primary and secondary turns. The leakage reactance x is more or less independent of the permea bility of the core, and consequently the transformation rateo of equation 6 is not a constant but depends upon the magnitude of the magnetizing current. Furthermore as x, decreases it necessarily approaches the value of r" and the error due to the increasing factor b becomes very large.

It is unfortunate that the saturation curve of the iron cannot be represented by a mathematical equation and therefore the only solution of the problem is the tedious step by step method. Although this method obviously is not absolutely correct, it does give a close approximation.

Consider first the problem where the secondary leakage reactance x is negligible. Then the change of flux need only



Sive the E.M.F. to force the secondary current through the resistance r*. Then

 $D d\Phi/d\theta = r^{n}i^{n} - r^{n} - r^{n}$

Representing do and r"i" in percent of maximum running condition

Or

 $d\Phi = r''i''d\Theta$.

Changeing from differential to difference, that is, replacing, as approximation, d by a gives

 $\Delta \Phi = r^{"}i^{"}\Delta \Theta .$

Using, finally, increments of △ e of 10°; then

$$\Delta \Phi = .175 i^{\dagger} r^{\dagger} - ... = ...$$

Summing up the 18 increments in 180°

$$\Sigma \Delta \Phi = \Sigma (.175 i^{"}r") = 2.$$

Or

$$\Sigma(i"r") = 11.43$$

Which value is the maximum point of the flux reached under normal condition. If the saturation curve is plotted so as to give the value of flux equal to 11.43 at maximum point in the running condition, and i" is expressed in percent of maximum normal current, then a current of 100% for 10° will necessitate a change of flux of one unit as plotted on the saturation curve.

The value of magnetizing current in percent of maximum normal current can be obtained from the stable condition of equation 11, when r", x", n" and n' are known, and the corresponding maximum flux taken from the saturation curve.

.

In this manner the flux curve may be calculated from the secondary current of any shape, and from which the magnetizing current may be calculated with the use of the saturation curve.

In many cases it is necessary to calculate the magnetizing current from the primary current, which necessitates an inverse process. As a first assumption the secondary current i" is taken equal to the primary current i' multiplied by the ratio of turns n''/n'. From this approximate value of i" the first approximate value of magnetizing current i is obtained, which value subtracted from the primary current and multiplied by n'/n" gives a second approximate value of the secondary current i", from which a very close second approximate value of i is calculated. The following example may illustrate more fully the method of procedure.

Example 3.

Assume the ratio of secondary to primary turns to be 10; the percent of primary magnetizing current to be .012; a residual magnetism in the core of .5 units and the accompaning saturation curve to be that of the primary coil. That is, in place of the value of flux at maximum normal condition being marked 11.43 it is given the value 11.43 n'/n" = 1.143.

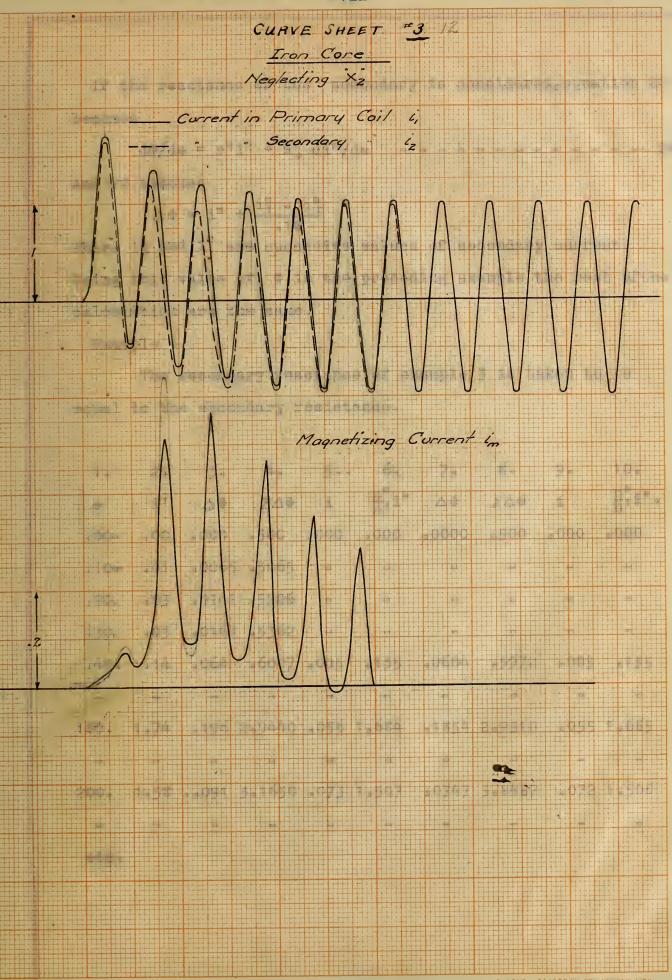
Then $\Phi = \mathbf{1}^n$.

The following tabulation is of a convenient form.

					(1,-6)					
1.	2.	3.	4.	5.		7.	₹.	9.	10.	
•	i*	ΔΦ	ΣΔΦ	i	$\frac{n}{n}$, in	ΔΦ	ΣΔΦ	i	$\frac{n}{n}$ "1"	
00.	•00	•000	•500	•000	•000	•0000	•500	•000	•000	
10.	•01	•001	•501	-	-	700		70	100	
20.	•03	•003	•504	•	an		•	-	-	
30.	•05	•005	•509	-	ه د	-		-	123	•
40.	•14	•014	•523	100		•		933	-	
50.	•25	•025	•548	-	100	-	-	-	85	
60.	•40	•040	-588	•004	•396	.0396	•588	•004	•396	V
70.	•55	•055	•643	•005	•543	•0543	•642	•005	•543	
90.	•67	•067	•709	.007	.663	.0663	.708	•007	.663	
90.	.80	.080	.788	•009	•791	•0791	•787	•009	•791	
100.	•93	•093	•880	•010	•920	•0920	•878	•010	•920	
110.	1.04	.104	.982	•011	1.029	•1029	•979	.011	1.029	
-	van	=	79	=	-	_	-	-	-	
180	1.74	•174	1.986	•020	1.720	•1720	1.981	•020	1.720	
un.	-	65	•	-	•	-	-	-		
540.	1.40	•140	3.75\$	-197	1.203	.1203	3 • 737	.187	1.213	
Column 2 is obtained from the orginal Surve of the primary										
current. The value of $\Delta \Phi'$ in column 3 is equal to n'/n" times										
the primary current i', and $\sum \Delta \Phi'$ in column 4 is the sum of $\Delta \Phi'$ in										
column 3 and the value of $\Sigma \Delta \Phi$ in column 8 of the preceding line.										
i in column 5 is found from the saturation curve for the value										
of flux corresponding to 4. n"/n' i" in 6 is equal to (i'- i) from										
which 7 is found and added to 8 of the preceding line and is										

the value of flux from which the final value of magnetizing

current i is determined. From column 10. i" no obsamed



If the reactance of the secondary is considered, equation 20 becomes

$$d\Phi/d\theta = r^n i^n + x_n^n di^n/d\theta$$
 - - - 24.

And 22 becomes

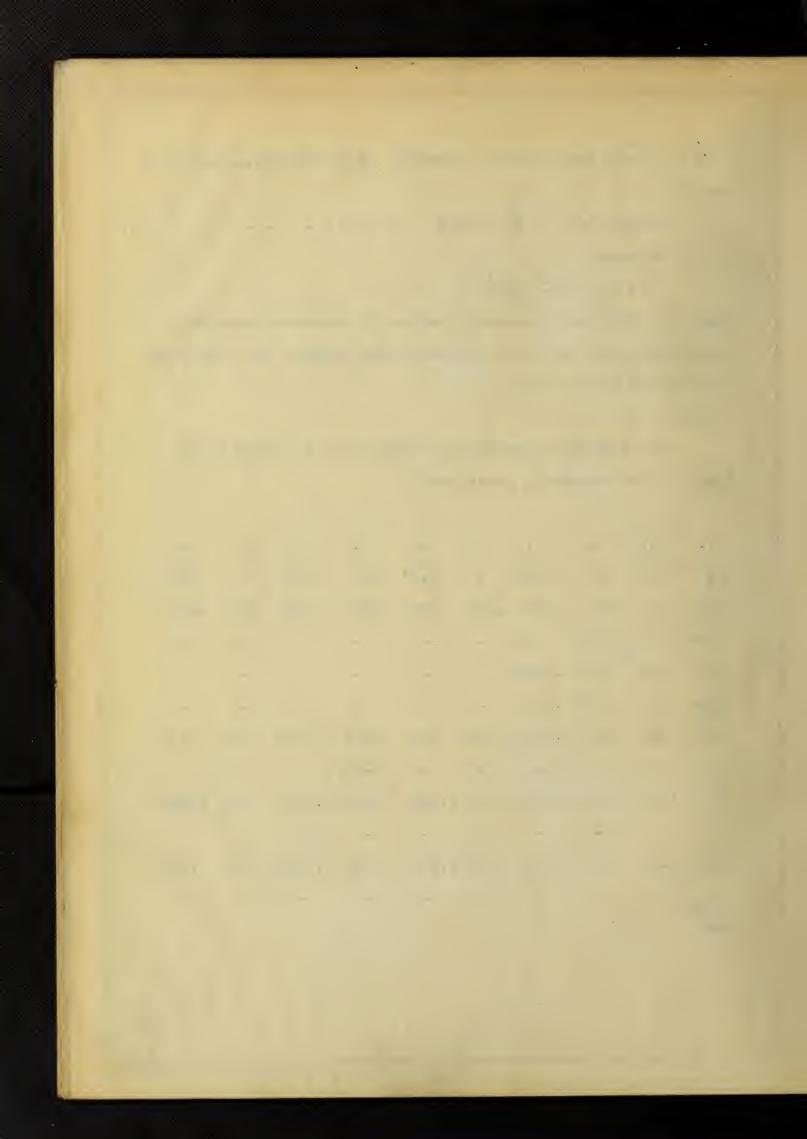
$$\Delta \Phi = i^n + \frac{i_n^n - i_2^n}{18}$$

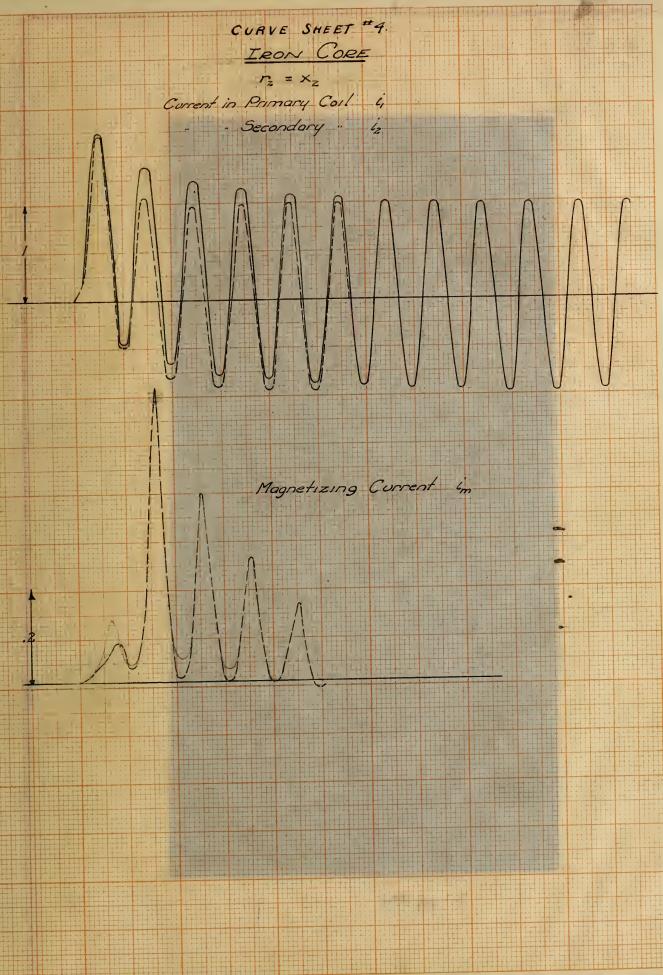
Where i and i are successive values of secondary current. Using this value of Φ in the preceding example the rest of the calculation are the same.

Example 4.

The secondary reactance of example 3 is taken to be equal to the secondary resistance.

1		2.	3.	4.	5	6.	7•	8.	9•	10.	
•	•	i,	$\Delta\Phi$	ΣΔΦ	i	$\frac{\mathbf{n}^n}{\mathbf{n}^n}$ i"	ΔΦ	ΣΔΦ	i	$\frac{n!}{n!}i!$.	
. (00-	•00	.000	•500	•000	•000	•0000	•500	•000	•000	
ø 1	10-	•01	.0065	•5065	-	-	•	-	-	•	
• 6	20.	•03	•0141	•5206	-	•	-	=	ca	es .	
4.3	30.	•05	.0161	•5362	**	•	=	-	-		
. 4	40.	•14	.064	.6007	.005	•135	.0604	•5972	•005	-135	
	•	-	-	•	-	•	es.	•	-		
180	0.	1.74	.196	2.9440	.056	1.684	.1854	2.9310	•055	1.685	
	-	-	-	-	-	-	-		=	53	
20	0.	1.58	091	3.1650	.073	1.507	•0747	3.1487	.072	1.508	
	-	-	. 🛥	-	-	-	•	•	•	•	
91	tc.										



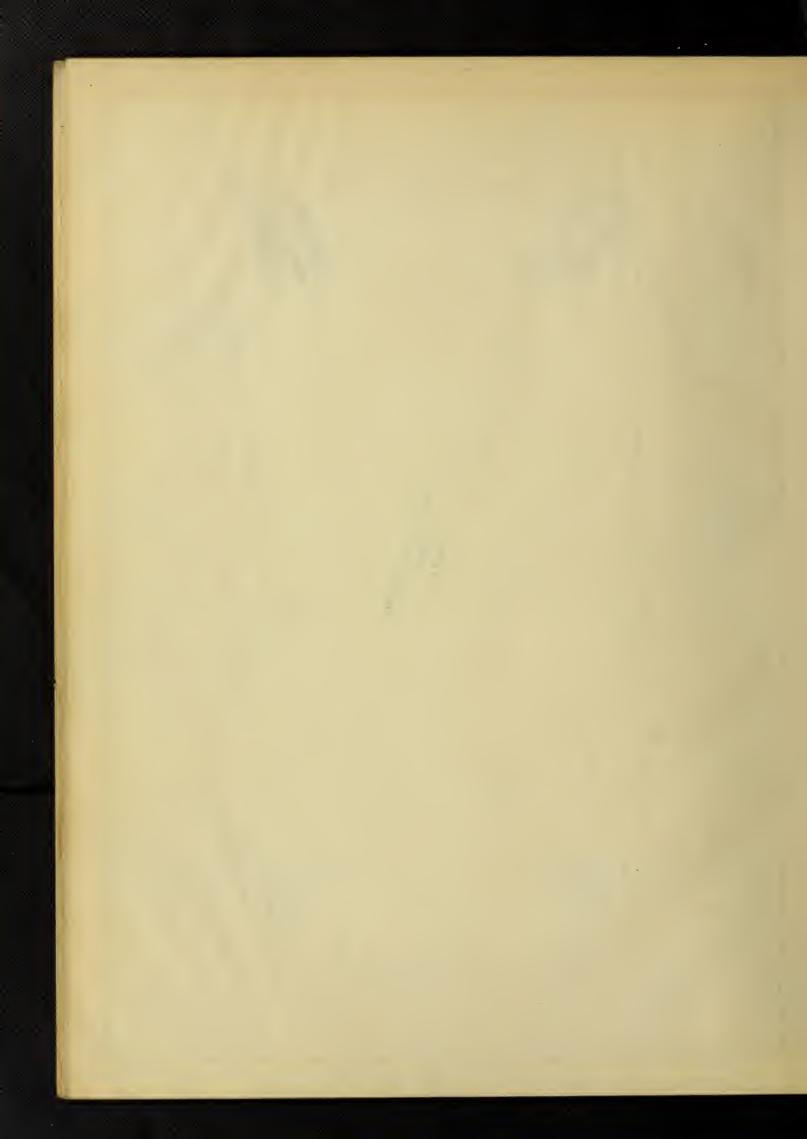


-14012 CURVE SHEET #4. IRON COEF Circut in Fundry Coil 4 Secondary. Magn fizing Current

Fing



su pg 15

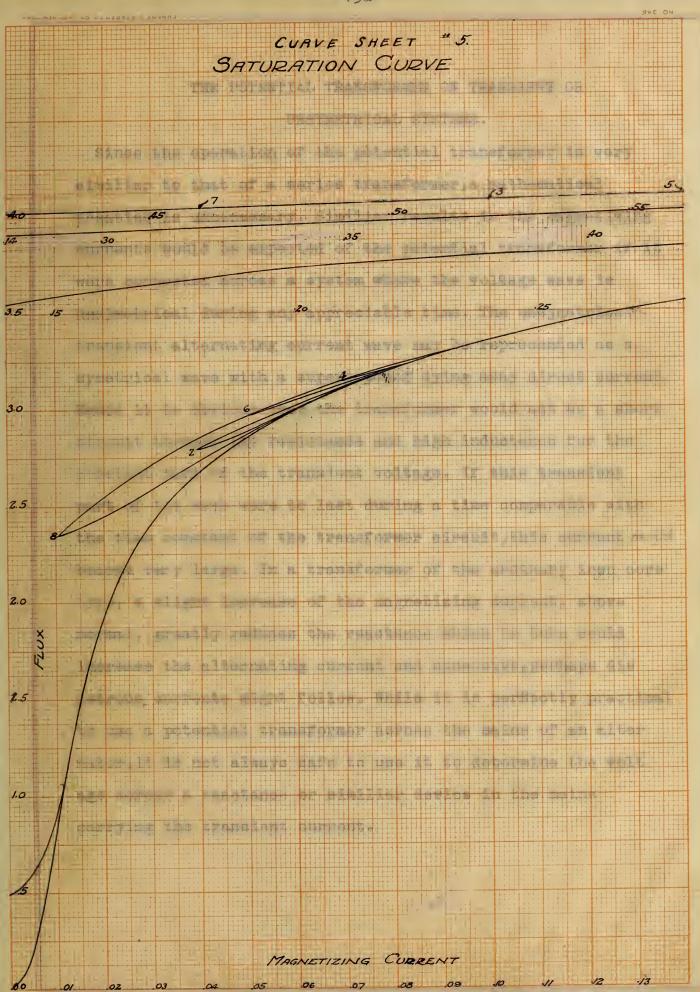


(pgs 124. 13a.

Curves 3 and 4, which correspond to examples 3 and 4, show the errors introduced by the use of the series transformer with iron core in recording transient phenomena. On comparing the two curves it is evident that the leakage reactance introduced in example 4 has a bad effect, which was not true in the case of the air core transformer. This is explained from the fact that the leakage reactance requires a larger magnetizing current and necessarily the flux works up to a higher point on the saturation curve.

The accompaning oscillograph record is that of the current through the secondary of the series transformer together with the current through the primary, which is the starting current of an inductive circuit with an electrical time constant x'r' of 10. These experimental curves are very similiar to those as calculated from example 3 and 4 although the constants are somewhat different.

The physical interpretation of the phenomena may be more clearly shown by the use of the accompaning saturation curve; where, the points marked with the odd numbers 1,3,5 and 7 are the successive maximum points of flux recorded, while the even numbers 2,4,6 and 8 are the successive minimum points. That is, the flux in the transformer passes over the curve from .5 to 1 back to 2,up again to 3 etc through the points 4,5,6,7 and 8 in succession.



CURVE SHEET # 5. SATURATION CURVE .50 40 35 MAGNETIZING COR ENT

THE POTENTIAL TRANSFORMER ON TRANSIENT OR UNSYMETRICAL SYSTEMS.

Since the operation of the potential transformer is very similiar to that of a series transformer, a mathematical treatise is unnecessary. Similiar results in the magnetizing currents would be expected of the potential transformer if it were connected across a system where the voltage wave is unsymetrical during any appreciable time. The unsymetrical transient alternating current wave may be represented as a symetrical wave with a superimposed dying away direct current. Hence it is obvious that the transformer would act as a short circuit through low resistance and high inductance for the constant part of the transient voltage. If this transient part of the wave were to last during a time comparable with the time constant of the transformer circuit, this current might become very large. In a transformer of the ordinary iron core type, a slight increase of the magnetizing current, above normal, greatly reduces the reactance which in turn would increase the alternating current and excessive, perhaps dis astrous, currents might follow. While it is perfectly practical to use a potential transformer across the mains of an alter nator, it is not always safe to use it to determine the volt age across a reactance or similiar device in the mains carrying the transient current.

SUMMARY

The following are the principal results that have been established by this investigation.

- 1. The series or potential transformer (especially with iron core) cannot be relied upon to record the instantan eous values of transient or unsymetrical systems.
- 2. An air core transformer(current) designed to record instantaneous currents, with a transient term, should have a very small secondary resistance and a large secondary reactance.
- 3. Great care should be taken in connecting a potential transformer across any part of a circuit carrying a transient current, so as not to have an unsymetrical voltage impressed across the transformer for any appreciable time.

